Tutorial 11
Advanced Graph Theory

October 29, 2013

1. In a large university with k academic departments, we must appoint an important committee. One professor will be chosen from each department. Some professors have joint appointments in two or more departments, but each must be the designated representative of at most one department. We must use equally many assistant professors, associate professors and full professors among the chosen representatives. (Assume that k is divisible by 3). How can the committee be found?

2. Several companies send representatives to a conference; the i^{th} company sends m_i representatives. The organizers of the conference conduct simultaneous networking groups; the j^{th} group can accommodate up to n_j participants. The organizers want to schedule all the participants into groups, but the participants from the same company must be in different groups. The groups need not all be filled. Show how to use network flows to test whether the constraints can be satisfied.

3. Let $[S, \bar{S}]$ and $[T, \bar{T}]$ be source/sink cuts in a network N.
    3.1 Prove that $cap(S \cup T, S \bar{\cup} T) + cap(S \cap T, S \bar{\cap} T) \leq cap(S, \bar{S}) + cap(T, \bar{T})$.
    3.2 Suppose that $[S, \bar{S}]$ and $[T, \bar{T}]$ are minimum cuts. Conclude from part-1 that $[S \cup T, S \bar{\cup} T]$ and $[S \cap T, S \bar{\cap} T]$ are also minimum cuts. Conclude also that no edge between $S - T$ and $T - S$ has positive capacity.
4. Prove or disprove: For every graph G, $\chi(G) \leq n(G) - \alpha(G) + 1$.
5. Prove that $\chi(G) = \omega(G)$ if G' is bipartite.